SOME BOUNDARY LAYER FLOWS OF NON-NEWTONIAN NANOFLUIDS OVER STRETCHING SURFACES

SYNOPSIS

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1 Introduction

The oldest physical branch of science is mechanics that describes the behavior of bodies at rest or in motion under applied forces. The study of objects at rest is termed as statics and study of objects which are in motion is termed as dynamics. Fluids (gases or liquids) are something which can deform due to the applied shear stress. In simpler words, a fluid is something which can flow and deforms with the force applied on it. Hence, composition of the words fluid and mechanics gave birth to an area known as fluid mechanics, which is related to understanding and controlling fluid behavior either under the state of rest or in motion. The study of fluids behavior at rest is called fluid statics and when fluid is in motion then it is termed as fluid dynamics. In this research work, we will discuss the behavior of fluids when they are in motion. Fluid dynamics finds its significant role in engineering fields, and industries such as oil reservoirs, chemical engineering, nuclear power reactor, biological systems, aerodynamics, power generations, and so on. In order to understand the behavior of fluids and their flows, it is essential to have a basic understanding of the fundamental equations for the flow of fluids [1].

The governing equations for the unsteady flow of viscous compressible fluids are:

• Equation of State

$$p = \rho RT,$$

• Equation of Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = 0,$$

• Equation of Motion (Navier-Stokes Equations)

$$\rho \frac{\mathbf{D}}{\mathbf{Dt}}(v_i) = \rho f_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\frac{\partial v_k}{\partial x_k}\delta_{ij}\right)\right],$$

• Equation of Energy

$$\rho \frac{\mathbf{D}}{\mathbf{Dt}}(C_p T) = \frac{\mathbf{D}p}{\mathbf{Dt}} + \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x_i}\left(\kappa \frac{\partial T}{\partial x_i}\right) + \phi_1,$$

Here, $\dfrac{\mathbf{D}}{\mathbf{Dt}} = \dfrac{\partial}{\partial t} + v_j \dfrac{\partial}{\partial x_j}$ is material derivative, $i, j = 1,2,3$ and

$$\phi_1 = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \frac{\partial v_k}{\partial x_k} \delta_{(ij)} \right) \frac{\partial v_i}{\partial x_j}$$

. Here v_i are the velocity components and x_i are the spatial components in x, y, and z-directions respectively, t is time, f_i are the body forces, κ is the thermal conductivity, C_p is the specific heat at constant pressure, T is temperature, μ is the dynamic viscosity, ρ is the density, p is the pressure, δ_{ij} is Kronecker delta, Q is the heat added per unit mass of fluid through conduction, R is the characteristic gas constant. Above equations for the unsteady flow of viscous incompressible fluids take the following form:

$$\begin{aligned}
\frac{\partial v_j}{\partial x_i} &= 0, \\
\rho \frac{\mathbf{D}}{\mathbf{Dt}}(v_i) &= \rho f_i - \frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 v_i}{\partial x_j \partial x_j}, \\
\rho C_p \frac{\mathbf{D}}{\mathbf{Dt}}(T) &= \frac{\partial Q}{\partial t} + \kappa \frac{\partial}{\partial x_i} \left(\frac{\partial T}{\partial x_i} \right) + \phi_1,
\end{aligned}$$

where

$$\phi_1 = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \frac{\partial v_i}{\partial x_j},$$

and $i, j = 1,2,3$.

On the basis of viscosity, fluids are further divided into two types, namely Newtonian fluids and non-Newtonian fluids. Sir Isaac Newton (1642-1726) studied fluids and proposed an important law for fluids which states that shear stress is directly proportional to the shear strain rate i.e. there is a linear relationship between the shear stress and shear strain. Mathematically it is given as:

$$\tau = \mu \frac{du}{dy},$$

where μ is dynamic viscosity and it remains constant at all shear rates at a constant temperature. The fluids which are following this law are termed as Newtonian fluids. Some fluids like water, oil, alcohol, gasoline, etc. exhibit constant viscosity and come under the category of Newtonian fluids. On the other hand, some fluids do not obey Newton's law, and in these fluids viscosity changes with applied shear. Scientifically, in non-Newtonian

fluid a single constant value of viscosity doesn't describe the flow properties of a fluid. All non-Newtonian fluids don't show the same behavior on the applied stress. Some are time-independent and some are time-dependent. Some fluids respond to the quantity of applied stress and others respond to the time taken for applied stress.

On the basis of time-independent and time-dependent fluids: The time-independent fluids are **Dilatants** (shear-thickening fluids), **Pseudoplastic** (shear-thinning fluids), and **Viscoplastic** (Bingham plastic). In dilatants, when stress is applied to the fluid, the viscosity increases. For example, cornstarch suspension, sugar solution, custard, etcetera. The viscosity decreases due to the stress applied to the fluid in Pseudoplastic fluids. These fluids are ketchup, latex paint, etc. In the case of viscoplastic, the fluid remains solid to some extent of force, but further increasing the stress (more than yield stress) it will deform. Toothpaste is the best example of these types of fluids. On the other hand, time-dependent fluids are Thixotropic and Rheopectic. The fluids like printer ink, ice-cream in which viscosity decreases and becomes thinner when shear is applied for a longer time are called **Thixotropic fluids**. The fluids like gypsum, POP in which the viscosity increases and becoming thicker when shear is applied for longer called **Rheopectic fluids**. Another category is the viscoelastic behavior of fluids. These types of fluids characterized by both fluid (viscosity) and solid (elasticity) like behavior under applied stress. The polymer melts, soap solutions, shows viscoelastic behavior under a specified range of shear rates [2], [3].

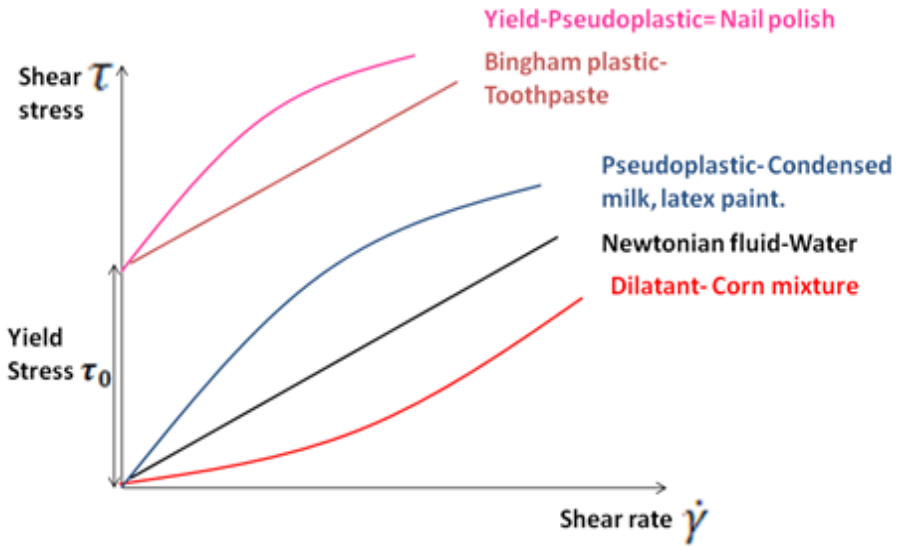


Figure 1: Classification of time independent non-Newtonian fluids

Non-Newtonian fluids show interesting behavior of fluids. These fluids are used to make body armor, which helps soldiers to move freely when not under attack and quickly becomes harder when a speeding bullet is confronted. There are many researches and applications of shear-thickening fluids [4] and other non-Newtonian fluids in our daily life, industries and many other fields [5]. In literature, many mathematical models [2], [3], [6]-[9] have been established for non-Newtonian fluids behavior. In time-independent fluids, let us start with the shear-thinning fluids, there exist many models for complexity variation of shear-thinning fluids. Some models are direct curve fitting and give the relationship between shear stress (apparent viscosity) and shear rate. Here more widely used models are discussed.

Non-Newtonian Fluid Models and Their Constitutive Equations

Power-law or Ostwald de Waele model: In this non-Newtonian fluid model the curve or graph between shear stress and shear rate is determined by this expression for representing the flow curve is

$$\tau_{yx} = m(\dot{\gamma}_{yx})^n$$

and the apparent viscosity μ(the ratio of shear stress to the shear rate) is given by

$$\mu = \frac{\tau_{yx}}{\dot{\gamma}_{yx}} = m(\dot{\gamma}_{yx})^{n-1}$$

Here, m and n are two empirical curve fitting parameters; m is the consistency index having units Pa.$(s)^n$(Pascal (second power n)), n is a behavioral index of fluid not having any units (dimensionless). In this model, the viscosity remains constant at a low shear rate μ_0 (having high value for given shear-thinning material) and also remains constant at a high shear rate μ_∞ (low value for given shear-thinning material) and within the range of shear rate, due to the increase in shear rate apparent viscosity decreases from μ_0 to μ_∞ On applying log, this model shows the straight-line graph between shear stress and shear rate within the limited range of shear rate $log(\tau_{yx}) = log(m) + nlog(\dot{\gamma}_{yx})$. So, this model is simple and very convenient to do calculations, simplifications etcetera. But this model also has disadvantages. It applies on a limited range only and then m and n also depend on the shear rate. It fails to provide information about shear viscosities μ_0 and μ_∞ The consistency index (m) dimensions depend on the behavioral index (n) values, so on the

different values of n comparison of m values is not possible and m values can be seen as apparent viscosity values. Despite these disadvantages, this is mostly used model in many engineering applications.

Carreau viscosity: This model is developed under consideration of molecular networks [10]. This includes the zero shear and infinite shear viscosity (μ_0 and μ_∞) in the improvement over the power-law model. So the model with included viscosities is given by

$$\frac{\mu-\mu_\infty}{\mu_0-\mu_\infty} = [1+\lambda(\dot{\gamma}_{yx})^2]^{n-\frac{1}{2}}$$

Here two curve fitting parameters are $n(<1)$ and λ; n is the behavioral fluid index (dimensionless) and λ is the time constant (relaxation time of the polymer in solution having time units). This model shows the shear-thinning fluid for wide range of shear rates but at the extended level of complexity added of parameters $(n,\lambda,\mu_0,\mu_\infty)$. This model reduces to Newtonian fluid behavior for n=1 and $\lambda = 0$.

Cross viscosity model: This model describes the behavior of shear-thinning fluids [11]. It has four parameters (n,k,μ_0,μ_∞). This model gained wide recognition, written as

$$\frac{\mu-\mu_\infty}{\mu_0-\mu_\infty} = \frac{1}{[1+k(\dot{\gamma}_{yx})^n]}$$

Ellis fluid model: This model is more relevant to use when power-law deviations are considerable at the low shear rate [12]. The mathematical expression for apparent viscosity in this model is

$$\mu = \frac{\mu_0}{1+(\tau_{yx}/\tau_{1/2})^{\alpha-1}}$$

Williamson fluid model: In this pseudo-plastic fluid model both maximum viscosity (μ_∞) and minimum viscosity (μ_0) are considered. Williamson [13] introduced a model equation for this fluid experimentally, written as

$$S = -pI + \tau$$

where $-pI$ shows the indeterminate part of the stress due to incompressibility constraint $(\tau = \left[\mu_\infty + \frac{\mu_0-\mu_\infty}{1-\Gamma\gamma}\right] A_1)$ and $(\dot{\gamma} = \sqrt{\frac{1}{2}traceA_1^2})$. Other symbols have the usual meaning.

By using binomial expansion, the stress tensor τ becomes

$$\tau = \mu(1+\Gamma\dot{\gamma})A_1$$

In time-independent fluids, the next non-Newtonian fluid category is shear-thickening fluids (dilatants). These types of fluids are exactly opposite to the shear-thinning fluids. So, the power-law model is also valid for shear-thickening fluids (dilatants). This power-law model on double logarithmic gave the linear relationship between shear stress and shear rate but it is not yet possible to ascertain whether these materials also display limiting apparent viscosities. The apparent viscosity, $\mu = m(\dot{\gamma}_{yx})^{n-1}$ for shear-thickening fluids increases when stress is applied and (behavioral fluid index is greater than unity $(n > 1)$.

Houska model: One model which is successfully describing thixotropic fluids [14]. The equation for the one-dimensional shear flow of this fluid is

$$\tau_{yx} = (\tau_{y0} + \tau_{y1}) + (m_0 + \xi m_1)\dot{\gamma}_{yx}^n$$

and $$\frac{d\xi}{dt} = a(1-\xi) - b\xi\dot{\gamma}_{yx}^{\varepsilon}$$

In the first equation, there are two contributions to this model. First is τ_{y0} (yield stress) and m_0(consistency index). Second is τ_{y1} and m_1 these depend on time contributions which are linearly dependent on 'instantaneous' structure which means ξ value. So, the first equation is true for constant ξ value and the second equation describes the rate of change of ξ with time where the first term indicates the build-up rate and the second term indicates the breakdown rate. The breakdown rate varies with the ξ values and shear rate values, and the buildup rate is affected by only the number of structural units at any instant time. In this model there are 8 parameters available; $m_0, m_1, \tau_{y0}, \tau_{y1}, n, a, b, \varepsilon$. Here $m_0, m_1, \tau_{y0}, \tau_{y1}, n$ shows material properties and a, b, ε parameters vary material to material [15].

Maxwell Model: Next category is the **viscoelastic** behavior of non-Newtonian fluids. Many mathematical models depend on the linear combination of viscous and elastic components. Maxwell model is well-preferred because in this model both viscous and elastic components relationship is correctly defined. Maxwell's mathematical model was obtained by a series arrangement of spring with Young's modulus (G) and dashpot indicating Newtonian fluid having viscosity (μ). The strain rates of spring and dashpot

are $\dot{\gamma}_1, \dot{\gamma}_2$ respectively and then total strain rate $\dot{\gamma}$ is given by

$$\dot{\gamma} = \dot{\gamma}_1 + \dot{\gamma}_2 = \frac{d\gamma_1}{dt} + \frac{d\gamma_2}{dt},$$

Now, multiply by μ (viscosity) on both sides,

$$\mu\dot{\gamma} = \mu\frac{d\gamma_1}{dt} + \mu\frac{d\gamma_2}{dt}$$

The first component of this equation shows elasticity and then includes Young's modulus (G). So, multiply and divides the first term by G, then the equation becomes,

$$\begin{aligned} \mu\dot{\gamma} &= \frac{\mu}{G}G\frac{d\gamma_1}{dt} + \mu\frac{d\gamma_2}{dt}, \\ \implies \mu\dot{\gamma} &= \lambda\frac{d\tau}{dt} + \tau, \qquad \lambda = \frac{\mu}{G}, \end{aligned}$$

This is because of Hook's law.

$$\begin{aligned} \tau &= -G\frac{dx}{dy}, \\ &= G\gamma_1 \\ \implies , \frac{d\tau}{dt} &= G\frac{d\gamma_1}{dt}. \end{aligned}$$

Therefore, $\mu\dot{\gamma} = \lambda\frac{d\tau}{dt} + \tau$ shows the model developed for viscoelastic fluids by Maxwell. In this model, if the stain rate $(\dot{\gamma})$ becomes constant then stress decays.

Heat transfer is the most significant phenomenon in nature, industrial and technology processes. The thermal properties of conventional fluids are poor than that of solids which results in a limited rate of heat transfer. Hence to ameliorate the rate of heat transfer some solid particles are added to the base fluids. Initially, for the modification of the thermophysical properties of the fluids, the millimeter or micrometer-sized [16],[17] particles were added into the base fluids which improves the heat transfer mechanism. Though these small-sized particles result in a lack of stability and lead to clogging and blocking. In order to break these shortcomings, nanomaterials (ranging from 1 to 100nm) are being used for the purpose to enhance the rate of heat transfer. Choi and Eastman [18] first started the idea of dispersion of nanoparticles in carrier fluids in 1995 and introduced the term called **nanofluids**. Nanofluids are the suspension of nanoparticles

in base fluids. The addition of nanoparticles enhances the thermal properties of the base fluid. The nanomaterials which are used for the preparation of nanofluids are non-metals, single or multiwall carbon nanotubes, and oxidized ceramics whereas base fluids can be water, glycol, oils, and lubricants. Below are some factors which affect the heat transfer rates in nanofluids:

Thermal conductivity

It is a measurement of the ability of a substance to conduct electricity. The thermal conductivity increases due to the addition of nanoparticles in the base fluid. In literature, nanofluids have higher thermal conductivity than that of base fluids [19], [20]. Some common methods used for the measurement of thermal conductivity are analyzer of thermal property [21] or hot-wire transient device [22]. After an extensive review, it was found that thermal conductivity depends on factors such as particles (size, shape, and material), base fluid material and temperature.

Particles size

Chopkar et al. [23] studied the effect of nanoparticles on the base fluids (water and ethylene glycol). Paul et al. [24] also discussed the effects of nanoparticles dispersed in the base fluid. These studies showed that with a decrease in the particle size the thermal conductivity increases.

Particles shape

Commonly, there are two types of particles used in nanofluids; cylindrical and spherical particles. Xie et al. [25] first discovered the effects of particle shape. Due to the larger distance ratio (length to diameter ratio) in the cylindrical particles, there is fast heat transfer which leads to improving the thermal conductivity as compared to that of the spherical particles.

Particle and base fluid materials

Nanofluids are made up of using different nanoparticles and base fluids. Nanoparticles as oxide ceramics, metal carbides, nitrides, metals, non-metals and base fluid as water, ethylene glycol and engine oil. According to the Maxwell model of thermal conductivity, it was concluded that the thermal conductivity of mixture as base fluid decreases, then

the thermal conductivity ratio (thermal conductivity of nanofluid to that of base fluid) increases.

Effect of temperature

The temperature also affects the thermal conductivity of base fluid and particles. Yu et al. [26] discussed the effect of temperature on the thermal conductivity of nanofluids having ZnO nanoparticles. Koley and Dey [27] also examined the effect of temperature on the thermal conductivity of CuO with gear oil nanofluids. These studies result showed that with the increase in the temperature of nanofluids, the thermal conductivity increases.

Viscosity

Choi and Eastman [18] considered that the viscosity of nanofluids would not be affected by the addition of nanoparticles, but this is not valid all the time. Viscosity is the internal resistance force offered by the fluids. After reviewing the literature, it is observed that viscosity depends upon factors such as volume concentration, shear rate, morphology and temperature which are discussed below:

Volume Concentration

In nanofluids, the most significant factor which affects the viscosity is volume concentration. Namburu et al. [28] examined the viscosity of nanoparticles of copper oxide, ethylene glycol and water. It has resulted that with the addition of nanoparticles, the viscosity of nanofluids increases. Sundar et al. [29] also concluded that the particle volume concentration improves the viscosity of nanofluids.

Morphology

Nanofluids morphology (shape and size) can also affect the viscosity. Nguyen et al. [30] showed that two particles of different sizes have different viscosity. Chevalier et al. [31] measured different nanoparticles (35, 94, 190nm diameter) of SiO_2/water and SiO_2/EG in different volume fractions and resulted that nanoparticles size decreases with the increment of shear viscosity.

Shear rate

Another factor which affects the viscosity is shear rate in non-Newtonian nanofluids. Halelfadl et al. [32] showed after investigated the viscosity of carbon nanotubes, for high

particles content nanofluids act as shear-thinning materials and Newtonian fluids for lower particle content. It was also found that the temperature did not alter the viscosity at a high shear rate. Yu et al. [26] discussed the influence of viscosity due to the shear rate. In this, the result found that low volume concentration and higher volume concentration of ZnO-EG nanofluids exhibit Newtonian and non-Newtonian behaviors respectively.

Temperature

Temperature is also a very important factor affecting the viscosity of nanofluids. Mena et al. [33] concluded that $Al2O_3$ – water nanofluids presented well within the range of temperature from 5 to 20 degrees Celsius and volume fraction of nanoparticles below 1 volume percentage. Turgut et al. [34] resulted that viscosity reduced with temperature. From all these studies it was showed that an increase in temperature decreases the viscosity. The nanofluids are the most attractive fluids for the heat transfer mechanism. Nanoparticles added in the base fluid enhance the thermal properties of the base fluid. Nanofluids have vast applications in coolants [35], power generators [36], reactors [37], and solar collectors [38]. In recent studies, nanofluids also including medical applications as cancer treatment drugs delivery [39]. To reducing the costs of production of nanofluids [40], [41] and improving the stability of nanofluids [42], [43] progressively efforts are made.

In the enhancement of the heat transfer mechanism, **Magnetohydrodynamics** (MHD) plays a vital role. MHD is the study of behavior and properties of electrically conducting fluid in the presence of a magnetic field. MHD study has been become an extensive research area due to its role in many fields of science and engineering. This study has imperative in making designs of MHD generators, solar power collectors, pumps and so on. The MHD flow is extremely important in planetary magnetospheres and chemical engineering aeronautics. Much information about aspects of MHD is also available in the book of Shercliff [44]. Gupta [45] firstly studied the electrically conducting fluid having steady flow in the presence of transverse magnetic current and he found that transverse magnetic decreases the heat transfer rate from the wall. The heat transfer and boundary layer flow over-stretching sheets have various applications in technology and industry, such as industries of papers, textiles, artificial fibers, continuous casting, rolling, rubber sheets, copper wires thinning and so on. Firstly, the study of flow over the linearly stretching sheet was specified by Crane [46]. Wang [47] was first investigated the flow hydrodynamics in

an unsteady stretching sheet of the liquid thin film.

In historical times, fluid flows were investigated assuming the fact that the fluid is inviscid which was not true at all because all these investigations gave absurd results. In 1904, Ludwig Prandtl determined the concept of the boundary layer. Prandtl showed that the small viscosity flow over the solid surfaces is divided into two regions: the region where viscous effects can be neglected (far from the surface) and other region where viscous effects can be considered (near the surface). A thin layer of flow nearby to the surface due to the friction between the fluid and solid surface is called the boundary layer [48]. The viscosity of fluid plays an important role in the boundary layer concept. Due to the viscosity variations in the motion of the fluid, there exists a significant condition known as the no-slip condition. This condition shows that the fluid adjacent to the solid body is at rest that's why the velocity is almost zero at the solid surface. At a certain perpendicular distance from the surface, effects of viscosity can be ignored and the velocity increases from zero (no-slip) to free stream velocity (frictionless flow) and this distance from the surface to fluid velocity reaches the 99 percent of free stream velocity is called boundary layer thickness. A detailed understanding of boundary layer flow is significant in processing applications because the flowing nature also influences the heat and mass transfer rates if there exists of temperature gradient [49], [50]. This theory started a new era in the history of fluid dynamics and became a powerful technique for understanding the compound nature of real fluids. The boundary layer theory used to facilitate the complex Navier-Stokes equations. These equations are elliptical in nature but the boundary layer equations are parabolic in nature, also Navier-Stokes equations signify the non-linear terms at higher values of Reynolds number which causes high complexity. For the two dimensional flow, the boundary layer equations (continuity and motion) for the unsteady incompressible flow of a viscous fluid in Cartesian component is written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}$$

with boundary conditions,

$$u = v = 0, \qquad \text{at} \quad y = 0,$$
$$u = 0, \qquad \text{at} \quad y \to \infty.$$

The boundary layer equations can be solved by a systematic (analytical) or mathematical (numerical) method. These analytical methods provide more accurate solutions for linear and less complexity problems and do not necessitate computation. These methods could not undertake more complex problems and then numerical methods are used. These numerical techniques are the approximate solution to highly complex problems and having the significant computational ability to solve problems with lesser time. Analytical methods for solving equations are homotopy analysis method, optimal homotopy asymptotic method, and spectral homotopy. Numerical methods suitable for the boundary layer equations are Runge-Kutta method, finite element method, finite difference method and spectral method. Both methods (analytical and numerical methods) available for solving boundary layer equations. Software like MATLAB, and MATHEMATICA is convenient for doing higher-order computations.

2 Literature Review

In fluid mechanics, the ancient achievements in research have taken place erratically. In the 18th and 19th centuries, fundamental discoveries started progressively. The earliest civilization had sufficient information to solve certain problems of flow such as sailing of ships with oars and irrigation systems. Archimedes (285-212 B.C.) gave the law of buoyancy and the parallelogram law for the addition of vectors and used these laws for the submerged and floating body. Leonardo da Vinci (1452- 1519) postulated mass conservation equations for one-dimensional steady flow and conducted test with the jets, hydraulic jumps, waves, etc. Sir Isaac Newton (1642-1727) stated motion and viscosity laws for linear fluids (Newtonian fluids). He was the first to assume the concept of frictionless flow. In the 18th century, many mathematicians (Joseph Louis Lagrange, Daniel Bernoulli, Jean D'Alembert, Leonhard Euler, and Pierre-Simon Laplace) found solutions for the frictionless flow of fluids. Leonhard Euler (1707 – 1783) postulated both differential and integral forms of the equation of motion. However, in practical problems

the assumptions of frictionless fluids have restrictions. D'Alembert showed that the body has zero drag in contact with the frictionless fluid and overcomes the limitations in perfect fluid assumption. The experiments on this theory gave rise to the growth of hydraulics and based on flow the data formed for open channels, waves, pipe flows and turbines.

At the end of the 19th century, the merger of experimental hydraulics and theoretical hydrodynamics has begun. William Froude (1810-1879) and Robert (1846-1924) have proposed the model testing laws, Lord Rayleigh (1842-1919) gave the dimensional analysis technique and Reynolds (1842-1912) has considered the experiment of classic pipe in 1883 screening the significance of the Reynolds number (dimensionless). Navier (1785-1836) and George Gabriel Stokes (1819-1903) added the Newtonian viscous terms to the governing motion equations, now called as Navier-Stokes equations. At last, Ludwig Prandtl (1875- 1953) considered the fluid flows with small viscosity like water and air flows gave the boundary layer theory. The boundary layer concept has overcome many mathematical problems of the viscous flows. Since the beginning of the 20th century, this theory has given significant encouragement to research into fluid mechanics. The brief history of boundary layer theory and numerical method (Keller box method) for solving boundary layer equations can be studied through the paper of Tani [51] and Keller [52].

Schowalter [53] and Acrives et al. [54] was firstly described the power-law fluid with the theory of boundary layer flow. It was concluded that the self-similar solutions exist for the boundary layer equations of power-law fluid flow when the flow is of Falkner-Skan type. Schowalter examined these boundary layer equations neglecting the body forces and also found the self-similar solutions in case of external velocity $U_e(x)$ is of form x^m, where x indicates distance along the body surface. Acrives et al. [54] examined the boundary layer flow in case of power-law fluid for $m = 0$ i.e. the flow along with the flat plate.

Sakiadis [55],[56] theoretically examined the boundary layer flow over continuous solid surfaces and compare the solutions with that of boundary-layer flow on finite length surfaces. The boundary layer differential and integral equations of momentum were derived for the surfaces. The result showed that the differential boundary layer theory and differential equations valid for the motion of continuous surfaces. But conditions of boundary were not the same for surfaces in both cases which means that the differential equations solution of motion will considerably different for two cases. He also examined

the laminar and turbulent flow equations for boundary layers on the continuous flat moving surface and cylindrical moving surfaces. The behavior of the laminar boundary layer was investigated by the numerical method (exact solution) and also by an integral method (approximate solution) and the behavior of the turbulent boundary layer was investigated by using an integral method only. In both cases of laminar and turbulent flow, he presented the boundary layer thickness equations, displacement thickness equation, momentum thickness equation and skin friction. The differences between boundary layer behavior on continuous surface and the finite-length surface were found. In case of the laminar boundary layer the drag on flat finite length was lower than that on continuous surface but more in case of turbulent boundary layer.

Bhatnagar [57] considered the two-dimensional flow of non-Newtonian fluid (incompressible flow) with an approximation of the boundary layer where viscosity and cross-viscosity is constant. It was concluded that with influence in pressure non-Newtonian boundary-layer equations reduce to that of Newtonian fluid, and pressure in the boundary layer of Newtonian fluid flow is less than that of corresponding non-Newtonian boundary layer having viscosity coefficient same. They also found that in both fluids velocity distributions remain identical.

Lee and Ames [58] considered the study of Eyring viscous and power-law fluids with boundary layer flow and derived the similarity solutions by using the method of group theoretics. In this study, similar differential equations for many flow systems were presented and they also discussed the similar solutions of a power-law fluid in different flow systems.

Tsou et al. [59] considered the continuously moving surface with the boundary layer flow and examined the flow and temperature fields analytically and experimentally in case of both laminar and turbulent flow. The results for the velocity of boundary layer, temperature distributions, coefficients of heat transfer and surface friction were given by analytical solutions. It was found that the experiments in laminar flow and good accord in measured velocity profiles with analytical predictions that analytically solutions of boundary layer flow over the continuous surface was physically realizable flow. The turbulent velocity profiles determined by experiments were also in good agreement with the analysis. The measurements of temperature distribution in the case of the turbulent

boundary layer exhibit suitable correspondence with the analysis. Moreover, the turbulent flow field analytical model was very well supported by the investigational information. The experimental findings of temperature distribution in the turbulent field were satisfactory in accord with the analytical predictions, however not as well held as the results in the velocity field.

Hansen and Na [60] considered the two-dimensional, incompressible, steady, laminar boundary layer flows for similarity analysis of non-newtonian fluids. The main result from the analysis that for any non-newtonian fluid model the similarity solution present in case of 90 degrees of wedge flow in which stress, strain rate associated with the continuous arbitrary function. They also represented the dimensionless form of the numerical solution of power-law Eyring fluids.

Thompson and Snyder [61] examined power-law fluid in the boundary layer flow with the injection of fluid at the surface to test out the drag reduction potential of non-Newtonian fluids. The solution varied for the range of injected mass of fluid and found that as the fluid index increases the coefficient of skin-friction monotonically decreases. However, in the case of a fixed rate of fluid mass injection, the reduction percentage in the skin-friction coefficient becomes more for less value of a fluid index. Also, further results concluded for various range of fluid injection rates and different fluid index values are briefly presented for variations in momentum and displacement thickness.

Thompson and Snyder [62] firstly investigated the two-fluid systems in boundary layer flow including the non-Newtonian fluid. They studied the boundary-layer flow of Newtonian fluid along with the flat plate on which non-Newtonian fluid is injected. In this case, the governing partial differential equations were reduced to ordinary differential equations by using similarity transformations and numerically solved by the shooting method. Finally, the result showed that if the fluid index remains constant and mass injection increases then the thickness of the boundary layer of lower non-Newtonian region increases. Reversely, if mass injection remains constant and the fluid index value increases then the lower region becomes thinner. In both cases, the outer Newtonian region became thinner.

Vleggaar [63] investigated the momentum and heat transfer to accelerating, continuous surfaces. The solution compared with that of boundary layers on the length of finite

surfaces and the continuous surfaces having a constant velocity. The conservation momentum and energy equations were given systematically. The governing ordinary differential equations were answered by using SYSSOL the computer program and he presented two applications firstly, cooling of the sheet (extended) secondly, heat and momentum transfer to monofilament (spinning zone part). In dimensionless quantities, on the flat surfaces, the momentum and heat transfer were high because of acceleration. However, in the case of cylindrical surfaces, the situation was complex due to the influence of the cylinder radius. It was found by comparing with the cylindrical continuous surface having constant velocity that the momentum transfer was higher in the case of an accelerating surface but showed lower transfer of heat.

Rajagopal et al. [64] studied the three-grade fluid with the incompressible flow of fluid over an infinite flat porous plate having suction. The existence theorem was established with the help of the shooting method. The perturbation analysis was used for investigating the problem of power. It was not obvious that the solution of perturbation converges and accordingly may not be a suitable answer for some range of material (not perturbation parameter). Finally, we use a numerical method by which the equation was integrated from infinity towards the plate using the Gears method and this provided a suitable solution for the problem.

Siddappa and Abel [65] studied Walters liquid B flow over a stretching sheet including suction. They obtained the exact solutions for the energy equation and for the equation of boundary-layer flow. They also obtained and analyzed the skin friction coefficient and boundary layer thickness expressions.

Timol and Kalthia [66] analyzed the similarity solutions for three-dimensional, laminar, incompressible boundary layer flows of non-newtonian (time-independent) fluids. From this analysis, the conclusion was found that the similarity solution exists for any model of non-newtonian fluids. The main limitation of this analysis was that stress and strain rate should be associated with arbitrary continuous functions.

Andersson and Toften [67] discussed the idea of obtaining numerical solutions of boundary layer flow for non-Newtonian fluids (power-law fluids). They gave a review of techniques for finding the solutions and shortened the tricks. Firstly, the governing PDE was reduced to ODE (non-linear) and then solved by the Keller box scheme. The

main findings from this result are rather than exactness in the power-law viscosity model, the Keller box method is more appropriate for solving the boundary layer problems of non-Newtonian fluids.

Dandapat and Gupta [68] discussed the boundary layer approximation of incompressible 2nd -order fluid flow over the stretching elastic plane surface. It was concluded that with the rise of the elastic parameter (k_1), skin friction reduces. The heat transfer analysis of this flow shows that if the ambient temperature and the wall are remaining constant then with the increase in k_1 (intended for fixed Prandtl number), the temperature increases at that point.

Hassanien and Gorla [69] examined the characteristics of heat transfer of incompressible, micropolar, steady fluid past over a stretching sheet (non-isothermal) and derive equations of the boundary layer for taken fluid on a stretching sheet with the help of micropolar fluids theory given by Eringen [70]. The flow of fluid and characteristics of heat transfer and significant parameter dependence were discussed by numerical solutions. The values of velocity, angular velocity, and thermal functions also presented. From this information, the factor of skin friction and the rate of heat transfer was evaluated. The heat transfer rate and friction factor were influenced by the mass transfer rate. Suction provides the temperature and velocity profiles in a uniform manner that decreases the gradients of velocity and temperature. The exponent of the power law of the temperature of surface significantly affects the heat transfer mechanism. The heat transfer rate increases with the increase in the Prandtl number. The thickness of the thermal boundary layer decreases with an increase in Prandtl number and as a result heat transfer resistance was reduced.

Hassanien [71] presented the flow and heat transfer over a flat continuous surface of the viscoelastic fluid (in a parallel free stream). This type of flow shows different boundary layer problems and solutions were different from that of flow on a flat finite length surface. That is, the flow on continuous surface solutions depends upon the relative velocity between the continuous plate and free steam and also on velocity ratio while in the case of boundary layer flow on a flat surface of finite length depends only on velocity difference. The velocity and temperature distribution profiles were obtained and tabulated in the range of viscoelastic parameters by numerical solutions. Also, the solutions for fluids flow and characteristics of heat transfer are examined by numerical solutions. For the

equal Prandtl, Reynolds numbers and velocity difference, the surface rate of heat transfer and surface friction was large in the case $U_w > U_\infty$ while compared to that of case $U_w < U_\infty$.

Pakdemirli [72] derived the boundary layer equations for power-law fluids by using a coordinate system to make these equations independent of the shape of the body immersed in the flow. These equations can be driven by using asymptotic expansions. It was concluded that power-law similarity solutions were the same as that of Newtonian fluids. The similarity solutions for stagnation-point and parallel flow over a plate were also presented. At last, shear stresses on the flat plate surface and different cases of stagnation point were calculated.

Wu and Thompson [73] investigated experimentally and numerically the Reynolds number effects on the lift and drag of flat plate in shear-thinning non-Newtonian (power-law) fluids. They predicted the coefficients of lift and drag with FEM code in good experimental results. Their work gave the Reynolds number values (as a function of the shear-thinning fluid index) below which approximation of the boundary layer may be inadequate.

Timoshin [74] examined the stability of non-Newtonian fluid with a boundary layer over the thin film of different fluids. He considered the two-fluid system and analyzes this by triple-deck theory. This work gave some attention to the instability that arises in the two-fluid flow systems and this approach was used to undertake a similar study in thin films of non-Newtonian fluids.

Hassanien et al. [75] examined the power-law fluid problem of heat and flow transfer over a stretching sheet having variable temperature of the wall by using boundary layer analysis. Firstly, the energy equation and Navier-Stokes equation were reduced by applying similarity transformation, then Chebyshev polynomial expansion used to solve the governing system of ODE (non-linear). The influence of Prandtl number, surface temperature, rate of mass transfer at the surface, flow index was investigated by parametric studies. It was concluded that heat transfer and friction factor depended upon the fluid parameters.

Ariel [76] discussed the laminar and steady flow of a liquid of second grade through a radially stretching sheet. In this study, it was shown that the boundary value problem arises due to the viscoelasticity of fluids and boundary conditions also arise due to the

differential equations order. It was concluded that for all viscoelastic parameter values the solution exists. Perturbation solutions for small viscoelastic parameters and asymptotic solutions suitable for large viscoelastic parameters are also obtained.

Yao and Molla [77] proposed a modified power-law for definite measurement of viscosities of non-Newtonian fluids also it is known as a physically realistic model. This model can be easily investigate boundary-layer flows by incorporating into boundary-layer equations and associated problems of heat transfer in non-Newtonian fluids. The non-removable singularity problems don't exist in the modified power-law correlation.

Xu and Liao [78] studied the heat transfer and boundary layer flow of non-Newtonian fluids (power-law fluids) past through a stretching sheet. They considered the velocity and temperature distribution of the stretching sheet in the form of $U(x) - Cx^m$ and $T_x = T_\infty + Ax^\gamma$respectively. The governed global self-similarity equations have been solved by the homotopy analysis method [79] to find the correct analytical approximations. This study gives a deep insight into the heat transfer and boundary layer flow for temperature and velocity distribution of the power-law fluid sheet.

Patil and Timol [80] considered the boundary layer flow of non-Newtonian fluid (power-law) for three-dimensional laminar unsteady flow past over a flat plate. The governing PDE and boundary conditions are transmitted to ODE (non-linear) by using two-parameter groups in a single step. This study concluded that the similarity solution conditions for the present flow situation are carefully drawn from the requirements of similarity conditions and then the most general form of similarity solution exists.

Khan and Hashim [81] studied the non-Newtonian Carreau fluids of two-dimensional flows for boundary layer equations of motion. Runge-Kutta with shooting method was used for the numerical solution of governing differential equations and graphical results formulated. This concluded that an increase in momentum thickness and a decrease in thermal boundary layer thickness were observed for the increasing values of the power-law index. It concluded that the fluid magnitude reduces for shear-thinning fluids and increases for shear-thickening with an increase in Weissenberg number. The coefficient of local skin friction decreases with an increase in *We* number. Also by increasing the Prandtl number, thermal boundary layer thickness and temperature decrease, and the magnitude of Nusselt number is depressed when *We* number is uplifted.

3 Research Gap Identification

It is evident from the literature review that extensive knowledge of boundary layer flow is significant in many ways and exposes it to higher limits in the future. Such kind of flows finds their plethora of applications in manufacturing and industrial processes such as annealing and thinning of copper wires and plastic films, materials which are traveling on conveyor belts, etc. In this research work, we will keep our focus on the understanding of boundary layer behavior of non-Newtonian nanofluids flows over stretching surfaces which is still the least explored area.

4 Objectives

The objectives of the present study are briefed below:

- To design computative models of the concerned problems by exploiting Buongiorno/Tiwari and Das nanofluids model.
- To utilize the similarity transformations in order to find the self-similar equations of the governing mathematical model.
- To exercise a numerical method in order to realize the numerical solutions.
- To set up a computational algorithm for the considered numerical method in MATLAB/ MATHEMATICA/ MAPLE.
- To understand the influence of physical parameters like magnetic field strength, porous medium permeability, chemical reactions etc. on the flow of distinct non-Newtonian nanofluid flows towards a stretching sheet.
- To represent these attained results in tabular and graphical forms.
- To comprehend the role of geometries (linear/ non-linear/ curved/ exponential) towards the flow of nanofluids.

5 Methodology

The dimensional boundary layer equations obtained from the physical problem will be transferred into the dimensional equations with the help of some transformations. After that, we will use an implicit finite difference technique to solve these highly non-dimensional boundary layer equations. In this research work, we will solve these

equations by using the Keller box method (KBM). This method has second-order accuracy as well as unconditional stability and is highly recommendable for the accurate solutions of parabolic differential equations. Following are the steps to find the solutions of the differential equations by using KBM:

1. Firstly, the governing differential equations will be reduced into a first-order system.
2. Secondly, the difference equations for obtained first-order system will be written with the help of difference approximations.
3. Thirdly, the resulting equations will be linearized by Newton's method (if they are non-linear).
4. Fourthly, the obtained system will be solved by LU decomposition.
5. Finally, results are plotted with the help of MATHEMATICA/MAPLE/MATLAB.

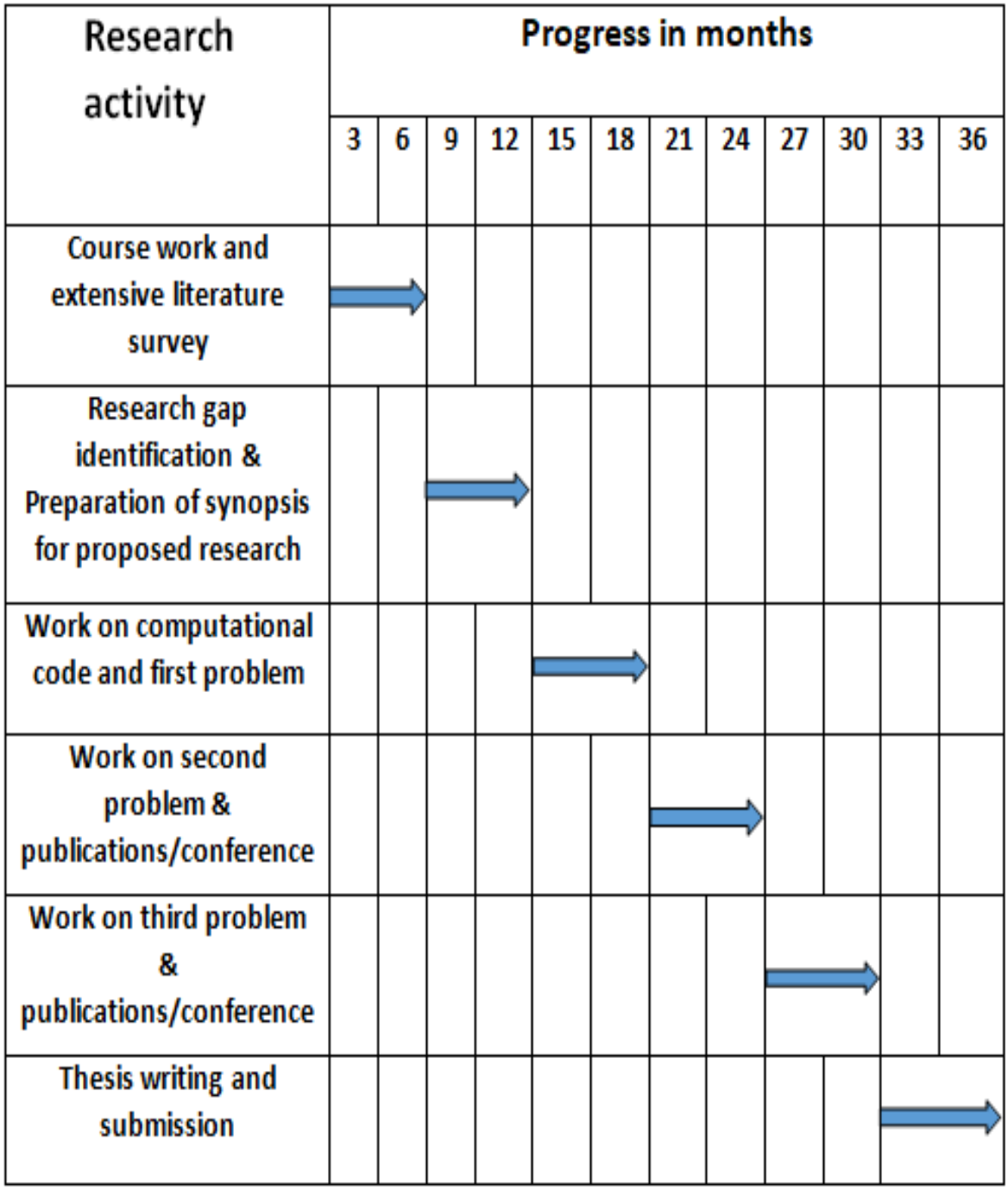
6 Thesis Outline

In order to achieve the objective stated above, the work epitomized in this thesis is divided into five chapters.

The tentative Chapterization is as follows:

Chapter-1	Introduction
Chapter-2	Literature Review
Chapter-3	MHD Boundary Layer Flow of Williamson Nanofluid Over a Stretching Sheet
Chapter-4	Boundary Layer Flow and Heat Transfer Characteristics of Micropolar Nanofluid Towards a Stretching Surface
Chapter-5	Flow and Heat Transfer Characteristics of Power-Law Fluids Past a Stretching Sheet
Chapter-6	Conclusion, Significance, Future Scope

Research plan schedule:



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